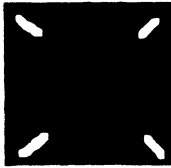


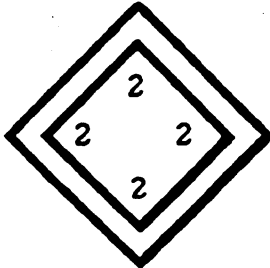
# Commentary

## Jupiter, I

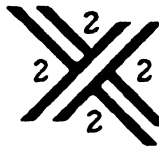
1. (a.7; b. 8; c. 3; d. 24) Students could practice making up their own Venn Diagrams about the class by picking characteristics such as eye color and hair color, or clothing combination. In this problem, the difficult part is (d) -- some students will try to use the numbers 7, 8, and 3 to get the total in the clubs.
2. (36) Angles have been identified in the figures.



4 right angles in the big black square



8 right angles in each white squares (16 total)



8 right angles at the intersection of the white squares (16 total)

3. (Monday) Students might make a list --S, M, T, W, T, F, S -- and start counting with Friday, till they get to 24.
4. (a. 149; b. 599; c. 30; d.  $3 \times n - 1$ ) The first two parts ask the student to notice that each second number is obtained by multiplying the first number by 3, then subtracting 1. Part (c) asks them to reverse this thinking, and part (d) asks them to generalize the pattern to any number  $n$ . The answer for (d) might be written in a number of different, equivalent ways.
5. (60 and 12) Students may use "guess and check" by listing the pairs of addends whose sum is 72; their guessing should get more precise as they get closer to finding the correct pair. They might get a hint as to where to start by noticing that the difference being 48 means that one of the numbers is above 50.
6. (d. \$3.18 ) The problem has students use their real-world number sense to get an answer.
7. (75¢) Three for 25¢ means that nine would cost 75¢; 10¢ each means that nine would cost 90¢.

# Commentary

## *Jupiter, II*

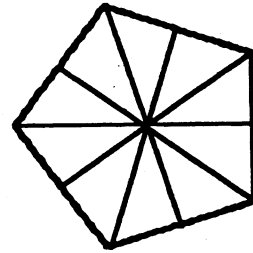
1. **(2 years)** One-half inch per month means 1 inch every 2 months. Students can therefore count month's "by twos" until they get to 12 inches. The count of 24 months is 2 years.
2. **(\$1.50)** Students at this grade level know intuitively that 50% is  $\frac{1}{2}$ , and they can find  $\frac{1}{2}$  of dollar amounts, usually without any actual computation.  $\frac{1}{2}$  of \$6 is \$3, and  $\frac{1}{2}$  of \$3 is \$1.50.
3. **(104, 68, 50)** The unusual thing about this pattern is that it's much easier if you start at the right end, and work to the left. You can see that you are adding 9 each step.
4. **(45)** Students will likely use a calculator to solve this problem. A few might notice that the sum of the first  $n$  counting numbers is  $n \times (n + 1) + 2$ . Therefore the problem becomes finding the first or smallest  $n$  such that  $n \times (n + 1) + 2 \geq 1000$ .
5. **(6:12 pm)** This problem involves elapsed time. Students can add 1:45 and 4:27, but they must remember that they aren't in the decimal system. They should get 5:72, and since 72 minutes is 1 hour and 12 minutes, 5:72 can be rewritten as 6:12.
6. **(Maria: 10; Patsy: 8; Colleen: 9; Kenyada: 11)** Students might make a list, or they may make name cards and act the problem out.
7. **(20 spaces ahead)** Each color should come up about  $\frac{1}{3}$  of the time. However, the orange moves and the blue moves cancel each other out, leaving about  $\frac{1}{3}$  of the time moving ahead 2 spaces.  $\frac{1}{3}$  of 30 spins is 10 spins, and at 2 spaces each move, you would be ahead 20 spaces.
8. **(She was wrong.  $x = 33$  grams)** Students can see intuitively that 1 block can be removed from both sides of the balance scale, leaving 3 sharpeners and 1 gram to balance 100 grams. Then the 3 sharpeners must weigh 99 grams, and then each would weigh 33 grams.  $x$  is used simply to introduce the idea of an unknown quantity as a variable.

## Commentary

### *Jupiter, III*

1. (The diagonal from upper left to lower right should be ringed.) Give students one star for having all the correct products in the chart, and another for the correctly-ringed diagonal.
2. (12) The ratio of 48 to 60 is the same as the ratio of 24 to 30, or 12 to 15, or 4 to 5. He would get the most bags possible by working with the 4 to 5 ratio, putting 9 items in each bag. This would give 12 bags, as  $12 \times 4$  is 48 and  $12 \times 5$  is 60.
3. (1:00) The only difficult part of this problem comes if students try to compute  $10:45 + 2:15$ , because they are not in the decimal system with time. The sum of 10:45 and 2:15 is 12:60, which is 1:00. Students with good number sense will likely "count on" from 10:45, using hours and then quarter hours.
4. 

Green	Black	Yellow	Students can be encouraged to solve such logic problems by making a chart, and proceeding by process of elimination.
Red	Blue	Orange	
5. (\$4) Students should have an intuitive feel for this type of problem, rather than subtracting \$11.15 from \$15.00, and rounding the answer. They should know that \$11.15 is close to \$11, and  $\$15 - \$11$  is \$4.
6. (a. 6; b. 63) 64 play, then the 32 winners of those matches play, then the 16 winners of those matches play, then the 8 winners of those matches play, then the 4 winners of those matches play and finally the last two winners play. This is 6 rounds of golf, and the winner must play in all of those. Since there are 63 losers, and each had to play a match to lose, there are 63 matches altogether.
7. There are 5 such lines of symmetry, as shown below.



8. (3,897) There are several clues that make this *guess-check-revise* problem a little friendlier. Since the sum of the four digits is 27, the average size of the digits must be fairly large. However, the *thousands* digit has to be either a 1, 2, or 3, while the corresponding *tens* digit is a 3, 6, or 9. Pick the 3 to begin the search, using 9 for the *tens* digit, and make the last digit a 7 since that's the largest odd digit not already used. This gives a sum of 27, as required, if 8 is the *hundreds* digit.

## Commentary

### *Jupiter, IV*

1. **(a. 70; b. 2520)** The student can multiply 14 times 5 for (a), and 14 times 180 for (b).
2. **(65° F)** Students can add 15 to 72, then subtract 22.
3. **(24)** Students may want to make a list and establish a pattern in order to solve this problem. They might name the pots shown as A, B, C, and D, and then see how many lists they can make, such as ABCD, ABCD, ACBD, ACDB, ADBC, ADCB. Those six are all the orders possible if A is on the left. There would be 6 such with B starting on the left, and 6 with C and 6 with D also, for a total of 24.
4. **(423)** *Guess-check-revise* is one way to solve the problem. A starting hint is that since the sum of the digits is nine, their average value is 3 so they are all small numbers.
5. **(Saturday)** Students might use calendar, or list S, M, T, W, T, F, S, and start counting with 7 on Tuesday, and count to 25.
6. **(a. 3 million; b. 36 million; c. 2 1/2 billion)** The problem situation calls for estimated answers rather than exact numbers, which would be misleading in such a problem. Students should be allowed leeway in their estimates, as they can vary quite a bit. Hopefully students will use a calculator to find (a), and continue to use it in finding (b) and (c) by entering only the non-zero digits to fit into the 8-digit calculator.
7. **(a. 10; b. 9; c. 9)** Students may use cubes or blocks to construct models. Students with good spatial visualization can find the answers from the pictures.
8. **(car and donkeys)** Students can approach this in a number of ways. Since the car matches 3 elephants from the second picture, they can be "removed" from the last tug of war without affecting the situation. Thus we are left asking which would win, 1 elephant matched against 3 donkeys. From the first picture, we see that an elephant pulls as much as 2 1/2 donkeys, so 3 donkeys would put pull one elephant. Therefore a car and 3 donkeys would out pull 4 elephants.

## Commentary

### Jupiter, V

1. (a. Answers will vary -- 10 and 11 are the most common answers; b. Answers will vary.) Students should use a calculator to compute:

$$25 \times 60 \times 16 \times 365 \times (\text{answer for part a})$$

If part a is 10, the answer is 88 million; if part a is 11, the answer is 96 million.

2. (11 quarters, 4 dimes) Some students will randomly use *guess-check-revise*, while others realize that the amount of money in quarters alone should be fairly close to \$3.15, and begin working backward from there, using *guess-check-revise*.
3. (rectangle: 28 cm; 2 triangles: 32 and 36 cm; 2 parallelograms: 32 and 36 cm) These are the four most likely answers, but a quadrilateral could also be built with a perimeter of 36 cm. Note: parallelograms cannot be named as rectangles.

4.

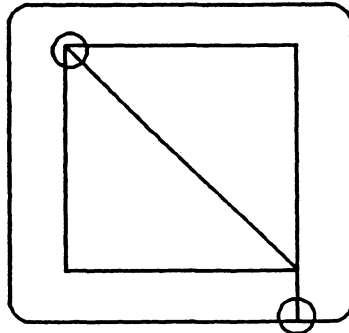
$$\begin{array}{r}
 4 \quad \boxed{5} \quad 6 \quad 8 \\
 \quad \quad 5 \quad \boxed{9} \quad 6 \\
 + \quad \boxed{5} \quad 9 \quad 4 \quad \boxed{7} \\
 \hline
 1 \quad 1, \quad 1 \quad 1 \quad 1
 \end{array}$$

5. (c. dime) A century is ten times a decade; likewise, a dollar is ten times a dime.
6. (12) Students have to consider a problem that is not one usually asked. If 3 is  $\frac{1}{4}$  of some number, what number is it?
7. (2 out of 3 chances, or  $\frac{2}{3}$ , or 67%) There are three spaces left, and two of those will result in a win for the computer. Any of the three spots are equally likely to be selected, so the chance is  $\frac{2}{3}$  of a win.
8. (25, 3, 3, 9) Students familiar with a Venn Diagram should have little difficulty with this problem. All the X's are counted for the first answer. Only 3 X's are in the RAP ring only. Three students are in the overlap between rock and country, but not in RAP. There are 9 students that are in the RAP and country circles together, but not in the rock circle.

## Commentary

Jupiter, VI

1. (28 hours, 30 minutes) Students will likely count from 7:15 one morning to 7:15 the next morning as 24 hours, and then count up by the hour to get to 11:15, finally counting a half hour to 11:45.
2. (770 feet) Students may draw the diagram and sub-divide it into two parts. Also, students can figure out the missing lengths.  $150 \text{ ft.} + 200 \text{ ft.} + 185 \text{ ft.} + 25 \text{ ft.} + 35 \text{ ft.} + 175 \text{ ft.} = 770 \text{ ft.}$  It is interesting to note that the perimeter of this figure is the same as if the figure were a 185 by 200 foot rectangle.
3. (a. \$33.10; b. 45; c.  $1/32$ ) The pattern for (a) is that each number increase by 20¢. For (b), each succeeding number decreases by half. Each next number in (c) is also half of the preceding number.
4. (B) Box A has a 3 out of 5 chance to win with red. Box B has a 2 out of 3 chance to win with red. If students change ratios so that they are based on the same second number, the result will be obvious. 3 out of 5 is the same as 6 out of 10 or 9 out of 15. 2 out of 3 is the same as 4 out of 6, 6 out of 9, 8 out of 12, and 10 out of 15. But then 10 out of 15 is a better chance than 9 out of ten. Students may run a probability experiment to verify this result.
5. (See figure below.) A network of paths such as the one below can be traced without lifting a pencil, if it has either 0 or two *odd vertices*. A vertex is *odd* if it has an odd number of paths going in or coming out. Furthermore, if you can trace the network, you have to start at one of the odd vertices, and you'll end up at the other. Therefore the two odd vertices circled below are the only places you can start, and trace the path.

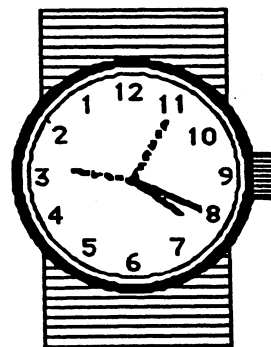


6. (7) This can be solved by guess-check-revise, or by working backward.
7. (a. 6; b. 3; c. 49) Students with good number sense will notice that the fractions involved are either close to zero or close to 1, which means that each mixed number would either be rounded to the whole number showing, or up to the next whole number. In (a),  $3 \frac{10}{11}$  rounds to 4 and  $2 \frac{1}{101}$  rounds to 2, so the sum is close to  $4 + 2$  or 6. In (b),  $5 \frac{2}{47}$  rounds to 5, and  $2 \frac{1}{35}$  rounds to 2, so their difference is close to  $5 - 2$  or 3. In (c),  $6 \frac{17}{19}$  rounds to 7, and  $7 \frac{3}{290}$  rounds to 7, so their product is close to  $7 \times 7$  or 49.
8. ( $1/6$ ) Students might draw a diagram to show that  $1/3$  of  $1/2$  is  $1/6$
9. (0) The ten one-digit numbers include zero, which makes the overall product zero also.

## Commentary

### *Jupiter, VII*

1. **(marble bag)** The chance of drawing a blue marble is  $\frac{1}{3}$ ; the chance of drawing a weekend day is  $\frac{2}{7}$ . We must compare these fractions to see which is larger. Finding a common denominator (21) allows us to compare the fractions by comparing the numerators.  $\frac{1}{3}$  is  $\frac{7}{21}$ , and  $\frac{2}{7}$  is  $\frac{6}{21}$ , and thus  $\frac{1}{3}$  is greater than  $\frac{2}{7}$ . Another way to compare the fractions is to use a calculator and change both fractions into decimals, and compare the decimals.
2. **(2000 years)** Many students will think you must multiply 4 and 2000, but the problem doesn't call for any computation if you think carefully about the situation.
3. **(25)** Students can use grid paper to make the rectangles that have 20 as a perimeter. The one with the largest area can then be found by counting unit squares.
4. **(To get back fewer coins)** Many people use a method like that mentioned to avoid carrying extra coins around in their pockets.
5. **(Juan is 15, Derrick is 5, Tyrone is 10)** A suggested strategy is to use *guess-check-revise* by guessing the youngest person's age, and doubling and tripling that amount to get the other ages, adding to see if the sum is 30. If not, revise the youngest person's age appropriately.
6. **(2:38; 2:57; 3:20; 3:48)** Students will have to either count backwards to get each new time, or subtract. Subtraction involves subtracting across non-base ten numerals.
7. **(See watch to the right.)** The time shown is 2:55, and adding 4:45 to that gives a time of 7:40. Showing 7:40 will be a challenge for many students, on this watch.



8. **( $\frac{1}{10}$ )** A quart is 2 pints, so 5 quarts is 10 pints. One pint is then  $\frac{1}{10}$  of 5 quarts.
9. **(a. answers will vary; b. answers will vary.)** Whatever a student writes in for (a), use a calculator to find 70% of that number by multiplication. Be lenient in checking accuracy -- give credit for being within one pound of the right answer for (b). Students will employ a variety of methods for finding 70% of their weight, if they don't use a calculator. Some, for example, might reason and take 7 out of every ten pounds they weigh, and then add on some extra for the pounds over a multiple of ten. Others might find 50% or 75% as those are intuitive numbers to work with ( $\frac{1}{2}$  and  $\frac{3}{4}$ ) for many weights, and then adjust their answer because 70% isn't exactly 50% or 75%.

# Commentary

## Jupiter, VIII

- (65) Students may use the *guess-check-revise* method. Some students might get the answer by putting the 36 and 94 on a number line, and deciding the point half-way between.
- |        |    |        |  |
|--------|----|--------|--|
| 6 , 4  | or | 4 , 6  | Perhaps the easiest way to solve each of these problems is to focus on the numbers that would give the indicated product, and then see which of those pairs of numbers would give the indicated sum. |
| 2 , 10 | or | 10 , 2 |  |
| 6 , 8  | or | 8 , 6  |  |
| 7 , 9  | or | 9 , 7  |  |
| 3 , 15 | or | 15 , 3 |  |
| 30 , 1 | or | 1 , 30 |  |
- (10) Students may act out this problem, or they might draw a diagram with A, J, S, C, and T around a circle. They would then connect each letter with each other letter with a line, and count the lines.
- (B) This is a two-step problem. Students will first have to find the sum of Karen's grades:  $92 + 88 + 99 + 97 + 89$  and get 465. Then they will divide 465 by 5 and come up with 93%, which is a B. Students can use a calculator for such situations.
- |               |   |
|---------------|---|
| 5 0 , 6 8 2   | The problem involves deducing the two missing numbers, and one way is to work through the standard subtraction algorithm for the numbers. |
| - 4 3 , 8 9 6 |   |
| 6 , 7 8 6     |   |
- (36<sup>0</sup> C) Students should realize that 12°C is too cold, and 120°F is too hot. Therefore by process of elimination, 36°C is the correct choice.
- (\$1. 16) This is a two-step problem. Students first have to decide how much Rachel spent. She bought 12 stamps at 32 cents each.  $12 \times \$0.32 = \$3.84$ . Next, the students compute what her change would be.  $\$3.84$  from  $\$5.00 = \$1.16$ .
- |           |  |
|-----------|--|
| 8 3 7 6   | Students can start by looking for the T value. Three such numbers must sum to give an 8 in the ones place; 6 is a good choice. Then knowing 1 is "carried" to the next place, then can solve for N. Proceeding in this way solves the problem. |
| 8 3 7 6   |  |
| 1 8 3 7 6 |  |
| 3 5 1 2 8 |  |
- (a. 70; b. answers will vary.) Part (a) involves multiplying 10 and 7. For part (b), whatever number the student puts in the first blank, divide the number by 7 in a calculator to get the number in the second blank. The answers will most likely be  $9 \div 7 = 1 \frac{2}{7} \approx 1.3$  or  $10 \div 7 = 1 \frac{3}{7} \approx 1.4$  or  $11 \div 7 = 1 \frac{4}{7} \approx 1.6$ . Be lenient in accepting reasonable answers for part (b), as some students will have the right idea but not know how to divide decimals or round their answers.

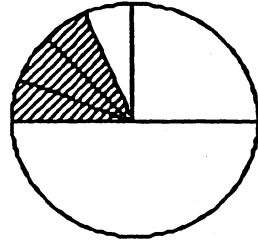


## Commentary

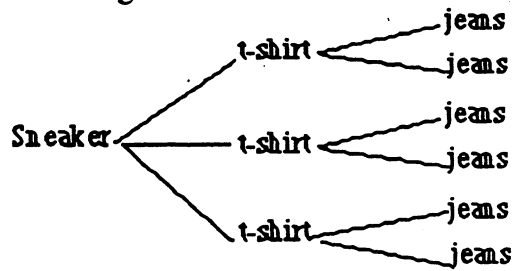
*Jupiter, IX*

1. (96) Students can count the cubes in layers. There would be 16 on each of the 6 layers, or  $16 \times 6$  total cubes.
2. (\$5) Students can compute 25% of \$10, 15% of \$10, and 10% of \$10 and add to get \$5 spent. Then  $\$10 - \$5$  gives \$5 left to spend. Another way is to add the 3 percents (25%, 15%, and 10%) to get 50% spent. Then 50% of \$10 was not spent, and 50% of \$10 is \$5.

3. (See diagram below. 3/16) Students might show the circle cut in half, then one of the halves cut in half to get fourths, then one of those fourths cut into four pieces, and three of them shaded (see below). If so, it would take 16 of the smaller pieces to make the whole circle, so each is 1/16. Three shaded sixteenths would be 3/16.



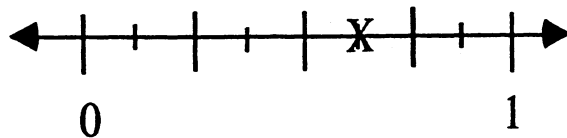
4. (6) One possible diagram is:



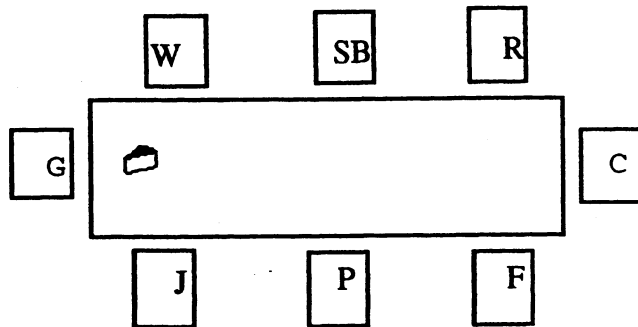
5. ( $3x - \$14.62$  or  $3 \times x - \$14.62$  or  $x + x + x - \$14.62$  or any equivalent expression)

6. (143) Students will add to find the answer.

7.



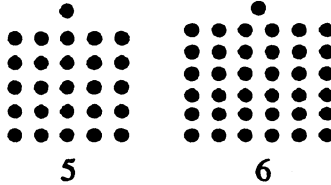
8.



## Commentary

### Jupiter, X

1. (See figures below) Note that each figure is a square with the same number of dots on each side as the figure number, plus an extra dot on top.



2. (a. 101; b. 20;  $n \times n + 1$ ) This problem encourages students to generalize the number of dots for each figure, rather than drawing them. Each figure is made from this number of dots: the figure number, squared, with 1 dot added on top.
3. (14) If the mother and one pup weighed 15 pounds, and the mother and two pups weighed 17 pounds, then the extra pup in the second weighing must be 2 pounds. Since all the pups are the same weight, 7 pups would weigh 14 pounds.
4. (See below.) There are other solutions. Students may use *Guess-Check -Revise*.

6	1	8
7	5	3
2	9	4

5. (168) Multiply 7 by 24.
6. (b) Students might use a calculator for this problem. For plan (a), you would earn \$55; for (b), you would earn \$102.30; for (c), you would earn \$60. Students are often surprised at how quickly an amount becomes, when doubled continuously.
7. (4)
8. (Mom) Students might take out a deck of cards and count the possibilities. Aces, face cards, and hearts when counted so they aren't counted twice, make up 25 of the 52 cards in the deck. The other cards, 2 through 10 of spades, diamonds, and clubs, would be 27 of the 52 cards. Since  $27/52$  is a better chance than  $25/52$ , Mom has a slight advantage. But not much.