Commentary

Earth, I

1. **(5 circles should be drawn in the right hand.)**

2. **(4 children)** There are 7 children who like chocolate and 10 who like strawberry. There are 4 children who like both chocolate and strawberry; they are in the overlapping part of the circles. Children might enjoy placing themselves in some loops like this made from rope, for other types of food such as spinach, beans and peas.

3. **(a. 50, 53, 54 ; b. 86, 84, 83; c. 25, 40, 45)** Give a star to a, b, and c separately. Note that (a) is simply counting from 48; (b) involves counting down from 87; (c) is counting by 5's, starting at 15.

4. **(even; odd; even; even; even)** This problem is a concrete introduction to *odd* and *even* numbers. Students might enjoy practicing this process with other numbers of coins.

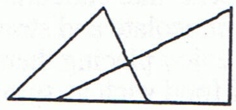
5. **(12)** The problem introduces students to the *repeating function* concept on a calculator. Most hand-held calculators will repeatedly add, subtract, multiply and divide in this manner. It is interesting for students to experiment with which number that is entered is the one that their calculator repeatedly uses. For the problem [5] [+] [3] [=] [=] [=] [=], for example, do they get 17 or 23?

6. **(4)** Many students will intuitively know that half of 8 is 4, so 4 squirrels went to get nuts. Thus 4 squirrels are left behind in the tree. If students have been taught a rule such as "how many are left means to subtract," they might not know how to solve this problem because there is no obvious number to subtract.

Commentary

Earth, II

1. **(5)** There are four small squares and one large square.Students may enjoy doing other problems of this nature, in which they find figures within other figures. For example, how many triangles are in this figure (3):



2. **(9)** 9 + 1; 8 + 2; 7 + 3; 6 + 4; 5 + 5; 4 + 6; 3 + 7; 2 + 8; 1 + 9.
3. **(12¢)** Two nickels and 3 pennies is 13¢, and the difference between 13¢ and a quarter is 12¢. Some students may have trouble with this problem if they don't know the value of the coins.
4. **(◆)** The pattern which repeats is ■◆◆♥♥♥. The fourth repetition of this pattern has started, and the first two figures are shown, leading to the third in the sequence as the one to follow.
5. **(a. 559; b. 850; c. 1,272)** Give a star for a, b, and c separately.
6. **(One possible answer is shown; there are other possible answers.)** Students may enjoy knowing that this is related to one of the "50 famous unsolved problems in mathematics" of the 80's. The problem was that everyone thought that such a map could be colored in four colors or less, so that no two boundaries the same color touched except at a point, but no one could prove it. Eventually the problem was solved, but for years and years, mathematicians enjoyed coloring maps like this, looking for an exception to the conjecture.

Commentary

Earth, III

1. **(4)** The bag needs to have four apples in it so that the scales will have the same weight on both sides. This assumes that all apples weigh the same. This problem is an important one to lay a concrete foundation for algebraic thinking.

2. **(8)** The student may think *what number, plus 9, equals 17.* Eight + 9 = 17 is part of a family of facts which also include: 9+ 8 = 17, 17 - 9 = 8, and 17 - 8 = 9.

3. **(a. 3 ; b. Pirates; c. 3)** The Hornets won 5 games; the Pirates won 4 games; the Eagles won 2 games; and the Bears won 1 game. For part a, the Hornets won 5 games and the Eagles 2 games, which is 3 games more. For part b, the Pirates won two more games than the Eagles. For part c, the student might want to get 12 pennies and move them around until he or she gets the same number in 4 different piles. If the student "even outs" 12 into 4 piles, he or she will get 3 wins; or said a different way: $12 \div 4 = 3$. This is a concrete introduction to the concept of getting an *average*.

4. **(a. 44; b. 32; c. 52)** The student may use "guess- check-revise" to find the answer by repeatedly trying different numbers for each box until they get one which works. Some students might realize that they can solve a different problem than the one given. For (a), they might solve by adding: $23 + \square = 67$; or they might solve by subtracting: $67 - 23 = \square$. Problems (b) and (c) can also be worked by solving a different problem.

5. **($1.28)** The student subtracts the value of the coupon, 25¢, from the cost of the apple butter, $1.53, giving $1.28.

6. **(4)** Purchasing 3 boxes of markers would provide 27 markers since 9 + 9 + 9 = 27. One more box is needed to give one marker per student, but 7 markers would be left over.

7. **(24)** The two insects would have 6 + 6 or 12 legs to offer to the collection. The three frogs would have 4 + 4 + 4 or 12 legs to add also. Therefore there's a total of 24 legs. This is a multistep problem which students can solve by drawing a picture of the frogs and insects, and counting legs. Or they might use the picture given in the problem, and count the legs that way.

Commentary
Earth, V

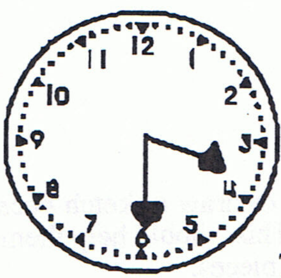
1. **(Thursday)** The students may make a calendar, starting with Wednesday the 8th, and putting the numbers in from 7 to 1, then from 9 to 16.

2. **(tape holder)** This problem will be difficult to many students who do not have an intuitive understanding of balance situations. It will be difficult for them to see that if 3 of object A weighs the same as 2 of object B, then B must be heavier. Actual balance scales in the classroom would help to see this inverse relationship between the number of objects to be a certain weight, and the weight of an individual object.

3. **(First: car; Second: van; Third: truck)** Students might act it out or find it helpful to write each word on an index card and move the cards around until each vehicle is in the correct order.

4. **(a. 23; b. 12; c. 38)** The student may use "guess-check-revise" to find the answers: 46 - 23 = 23; 30 - 18 = 12; and 24 + 14 = 38. They also might do different problems from the ones given, by solving a related problem such as 46 - 23 = ☐ or 23 + ☐ = 46 for (a), and so on for (b) and (c).

5.



The student should understand the hour and minute hands on a clock. The hour (shorter) hand should be between 3 and 4; the minute (longer) hand should be on the 6.

6. **(a. 18; b. 8; c. 8)** This problem is related to Venn diagrams, which students have likely met in first grade. They may need to be reminded that numbers can be in more than one figure. For part a, the numbers in the rectangle are 8, 9, and 1: 8 + 9 + 1 = 18. For part b, the numbers in the rectangle and circle are 2 and 6: 2 + 6 = 8. For part c, the numbers in the rectangle and not in the circle are 5 and 3; 5 + 3 = 8.

7. **(40¢)** This problem can be solved in 2 steps, by adding the two numbers and subtracting their sum from 79¢, or by subtracting one number from 79¢ and then the next number from what is left. In either case, the answer is 40¢.

8. **(about \$5)** Students should realize that 95¢ is close to \$1, and that there are 5 school days in a week. Therefore it will cost about \$1 a day for five days, or about \$5 for lunch at the school for a week.

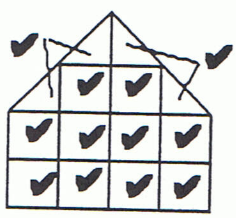
Commentary

Earth, VI

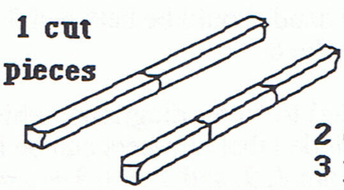
1. **(15)** Students might first add 9 and 12 and then subtract 6, or they might realize that only half a dozen, 6, need to be added to 9. Some students might not know what a dozen means, but having the egg carton shown should be a hint. Most students will intuitively know what "half" means in this situation, and can count half the eggs shown for "half a dozen."

2. **(a. L, N; b.**  **,**  **; c. 54, 49)** In pattern a, the pattern skips one letter each time. In pattern b, the dog, pencil, dog, cake pattern repeats. In pattern c, 5 is subtracted from the previous number each time. In the last pattern, some students might get the answer by the rhythmic count of numbers that end in 9 followed by numbers that end in 5, working backward through the decades.

3. **(12)** The concept of area in this problem includes "half-squares." It is helpful for students to use figures where the halves fit together to make another whole unit square rather than counting "half" each time. In the figure given, each ✔ is one whole unit square.



4. **(20 minutes)** It might be helpful for students to act it out, or draw a sketch because some might think that two pieces would require two cuts. This should help them see that only two cuts are required, at ten minutes each, to get three pieces.



5. **(Annie: 25; Baldwin: 34; Carl: 18)** Students may use "guess-check-revise" or logical reasoning to solve this problem. If the boys have *even* numbers on their shirts, Annie must have the only *odd* numbered shirt. Baldwin's number must be even and have a sum of seven; the only number with these characteristics is 34. Carl's number then must be 18.

6. **(clown: (5,2); train: (2,1); elephant: (3,4))** It is important for students to realize to go east (right) first, then go north (up) to locate points. For students having trouble, have them trace the path with their finger.

Commentary

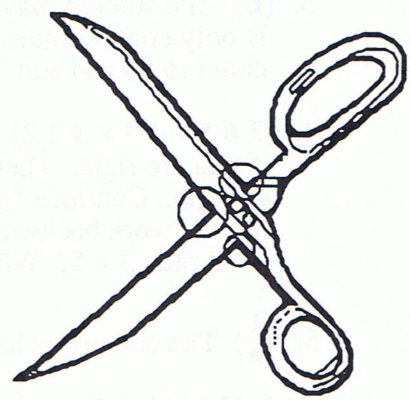
Earth, VII

1. **(90¢)** Students may identify the pattern as "adding on" 15¢ each time.

2. **(a. TV; b. sleeping; c. eating)** Students have an opportunity to work with a circle graph to answer each question. The answers are based on visual estimates of the size of one region as compared with another.

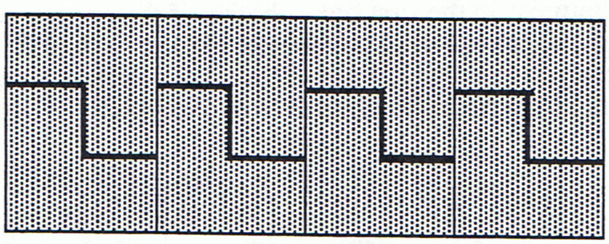
3. (>) 28 > 27

4. **(See below.)** The drawing to the right has several angles circled. Be a little generous with checking the paper. For example, if students circle a sharp point of the scissors, give them credit although technically part of the tip has a curved edge.



5. **(7)** Starting with the first clue and proceeding in order, the only numbers whose sum is 3 are 1 and 2, so mark them out. The only 2 numbers left whose sum is 8 are 3 and 5, so mark them out. The only 2 numbers left whose sum is 12 are 8 and 4, so mark them out. The only 2 numbers whose sum is 15 are 6 and 9, so mark them out. Seven is left

6. **(8)** Students might be encouraged to cut out a shape like the one shown, and physically move it around the grid to cover it. Such an arrangement is shown below.



7. **(5 centimeters)** Students might take a piece of string and curve it to fit the mouse's tail, and then measure the string. Or they might try measuring with a straight-edge centimeter ruler -- if so, they might have selected 2 as the estimate unless they somehow "go around the curve" in small chunks. 10 and 13 centimeters should be obviously wrong.

Commentary
Earth, VIII

1. **(3)** January 21st is a Monday. Three Sundays have already passed in January: January 6, January 13, and January 20. The student can locate January 21, move backward a space to the Sunday column, and count backwards three Sundays in that month.

2. **(28)** The student must know what "triangle" means, and also know that there are "overlapping" triangles in the drawing. There are 8 triangles in the cat's head --each eye contains 3; 13 triangles in the cat's body, and 7 triangles in the cat's tail: 8 + 13 + 7 = 28 in the entire body.

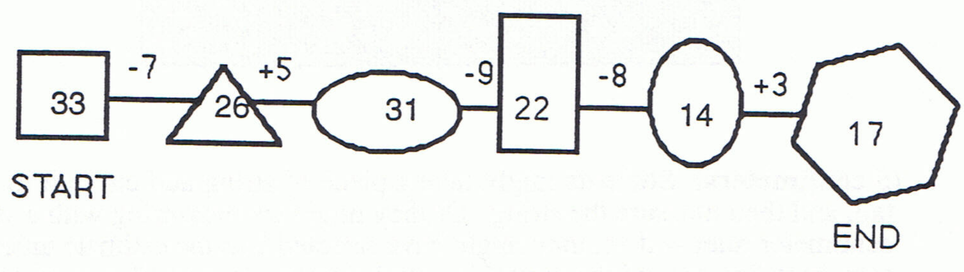
3. **(B)** The student may look for a pattern in several ways. The student may observe that column B only contains numbers that end in a 7 or a 2. Or a student may look at column E, mentally count to 50 and add "2 more." Or the student may complete the chart to make a list.

4. **(3 6 5 - 4 3 = 3 2 2)** Start in the ones column. "Guess" a number and then "check" to see if you are right. Then go to the tens column and "guess and check." End in the hundreds column. Continue "guessing and checking" until you find the right number. Or the student might "work backwards" by turning the subtraction situation into an addition one; for example, what plus 2 = 5? What goes in the box must be 3. Continue in this fashion.

5. **($\frac{1}{4}$)** The car covers half of the circle; the robot and the telephone each cover $\frac{1}{2}$ of the half that is left, or $\frac{1}{4}$. The chance of landing on the telephone would be "1 out of 4," or written as a fraction: $\frac{1}{4}$

6. **(50 feet)** It might be helpful to draw a picture. By drawing one "pole" or "dot" and then continuing until a total of 6 are drawn, one can understand that there are five spaces between the six poles. Each space is 10 feet, so 10 + 10 + 10 + 10 + 10 = 50 feet in all.

7. **(33, 26, 31, 22, 14 go in the shapes.)** The problem can be solved in several ways In "guess-check-revise," try a number in the first box and calculate across; if the ending number is not correct try another number in the first box -- higher if the answer was too low, and lower if the answer was too high. Continue until the correct number is found. Or work backwards, by starting with the number you know, 17, and asking what number, when added to 3, gives 17? The number is 14. Continue working backward from the right end to the left end, in this fashion.



Commentary

Earth, IX

1. **(6 hours)** It is helpful to draw a picture of the lizard's trip. At hour 1, the lizard started at 0, went up to 2, and down to 1. At hour 2, the lizard started at 1, went up to 3, and down to 2. At hour 3, the lizard started at 2, went up to 4, and down to 3. At hour 4, the lizard started at 3, went up to 5, and down to 4. At hour 5, the lizard started at 4, went up to 6, and down to 5. At hour 6, the lizard started at 5, went up to 7, and climbed out!

2. **(4 hours)** The essence of this problem is to know that Howard watches T.V. from 11:15 to 12:15, from 12:30 to 1:30, and from 5:30 to 7:30. The first and second times he watched for an hour each, and the third time for 2 hours, totalling 4 hours in all.

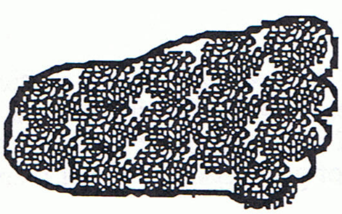
3. **($10)** A student can use a calculator, but many won't need one. Intuitively they can add $2.50 to itself to get $5, twice, and $5 + $5 is $10. It would be interesting to see the other strategies that students use on this problem.

4. **(b. less than 50 grams)** If the hot dog and bun were exactly 50 grams, the scale would be even. Since the 50 gram weight is lower it must be heavier. Therefore, the hot dog and bun must be less than 50 grams.

5.

The pattern repeats after every four figures. The 15th figure, then, will be identical to the 3rd figure. Some students will recognize this, but some may need to draw each figure out to the 15th.

6. **(accept between 13 and 21 as an answer.)** The figure below shows 15 thumb prints, which cover the footprint but with some "holes." The problem should encourage estimation since an exact answer can't be obtained by the students.



7. **(16)** Since students do not know how to divide yet, they will try a number of different strategies to find the answer. One is to ask yourself "what number taken twice will give a sum of 32?" Students might try a few numbers and see.

Commentary

Earth, X

1. **(1979)** Subtract 16 from 1995 and get 1979. Or, a student might be successful by counting backwards 16 times from 1995.

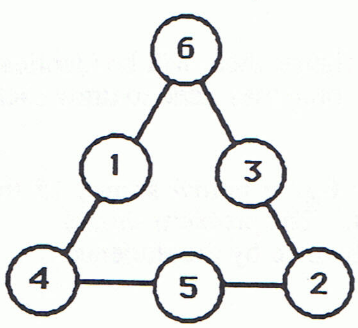
2. **(6,528)** This problem can be solved using the "guess-check-revise" method, using the numbers given: 2, 5, 8, and 6. Students might put these four numbers on index cards, and physically move them around until they find the right combination. The number has to begin with 6, so that card would stay stationary while the student moves the other three.

3. **(a. 7; b. 15)** In part (a), note that the graph is shaded halfway between 6 and 8, so there are 7 that like raspberry. In part (b), 5 second graders like lemon and 10 like strawberry for a total of 15. Students who have not seen graphs in which all the numbers are shown on the bottom axis might have difficulty with the problem for that reason.

4. **(9 and 12)** There are several ways to approach this problem. One way is to use "guess and check" until you have the correct pair of numbers. Another way is to make a list of pairs of numbers that equal 21:

   1 + 20 4 + 17 7 + 14 10 + 11
   2 + 19 5 + 16 8 + 13
   3 + 18 6 + 15 9 + 12 -- only this pair differs by 3

5. **(see below)** Some students may use " guess and check" until they find the right combinations for 11. Note that the three sides of the triangle can be switched around.



6. **(c should be circled)** Notice that the last figure has 4 vertices (points where the paths meet), and each has an even number of paths coming out of it. Networks such as these are *traceable* if they have exactly 0 or 2 odd vertices. This network has 0 odd vertices, since all four vertices have an even number of paths coming out of them.

7. **(6)** Students can see that a stapler weighs 12 grams from the smaller scale. Therefore on the larger scale, the two staplers weigh 24 grams. Since the entire weight is 30 grams on the big scale, the ball must weigh the difference between 30 and 24, which is 6.

8. **(15 + 8 = 23 or 18 + 5 = 23)** Students can again take five index cards, but this time label them =, 5, 1, 8, and +, and arrange them to give 23 as an answer.